Late-time symmetry near black hole horizons

Kentaro Tanabe,[1] Tetsuya Shiromizu,[2] and Shunichiro Kinoshita[2]

[1] *Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*
[2] *Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

It is expected that black holes are formed dynamically under gravitational collapses and approach stationary states. In this paper, we show that the asymptotic Killing vector at late time should exist on the horizon and then that it can be extended outside black holes under the assumption of the analyticity of spacetimes. This fact implies that if there is another asymptotic Killing vector which becomes a stationary Killing at a far region and spacelike in the "ergoregion," the rotating black holes may have the asymptotically axisymmetric Killing vector at late time. Thus, we may expect that the asymptotic rigidity of the black holes holds.

I. INTRODUCTION

Black holes in our Universe are expected to be formed under gravitational collapses, and to finally approach stationary and vacuum states by radiating and absorbing energy, momentum, and angular momentum. Then the uniqueness theorem [1] guarantees that the black hole candidates in our Universe are the Kerr black holes. The key ingredient for the proof of the uniqueness theorem is the rigidity theorem [2–5]. The rigidity theorem shows that the stationary rotating black holes have axisymmetric Killing vectors. The outline of the proof is as follows: The stationarity of spacetime implies that there are no gravitational waves around a black hole. Then the expansion and shear of the event horizon must vanish by virtue of the Raychaudhuri equation and stationarity. Using the Einstein equations, then, we can find that the null geodesic generator of the event horizon is a Killing vector. If the black hole is rotating, this new Killing vector may deviate from the stationary Killing vector which becomes spacelike in the ergoregion. This means that the stationary black holes might have two Killing vectors, that is, the stationary and axisymmetric Killing vectors. Hence the stationary black holes should rotate rigidly.

However, the late-time phase of black holes produced by gravitational collapses would not be exactly stationary but nearly stationary. "Nearly stationary" means that the black holes are surrounded by the gravitational waves at late time. Then we cannot apply the rigidity theorem to such black holes because the expansion and shear of the null geodesic generator on the event horizon do not vanish due to the presence of the gravitational waves on the event horizon. Note that the late-time behaviors of the perturbations around the Schwarzschild and Kerr black holes were examined and it was shown that the perturbations decay at late time both on the horizon and null infinity at the same rate (for examples, see Refs. [6–8]). In the dynamical processes in gravitational collapses, however, it is quite nontrivial whether the formed black hole approaches the Kerr black hole. In this paper, we show that the asymptotic Killing vector, which will asymptotically approach a Killing vector at late time, should exist on the horizon without assuming any symmetries and then that it can be extended outside of the event horizon using the Einstein equations. This indicates that if there is another asymptotic Killing vector which will be an asymptotically stationary Killing vector at a far region, the rotating black hole may have the asymptotically axisymmetric Killing vector at late time.

This paper is organized as follows. In the next section we provide the Einstein equations near the future event horizons. In Sec. III, we investigate the late-time behaviors of the metric on the event horizon and find the late-time symmetry. Then, under the assumption of the analyticity, we show that this late-time symmetry can be extended outside of the event horizon using the Einstein equation. In Sec. IV, we provide a summary and discussion. In Appendix A we perform the $(n + 1)$ decompositions of the Einstein equation. Using them, we provide the explicit form of the Einstein equation in the current form of the metric in Appendix B. Almost a similar formulation is developed in the study on null infinity [9]. In Appendix C, we present the details of the discussion associated with gauge freedom.

II. BONDI-LIKE COORDINATE AND EINSTEIN EQUATIONS

In this section, introducing the Bondi-like coordinate near the event horizon, we investigate the initial value problem. For the details of these derivations, see Appendixes A and B.

A.   Bondi-like coordinate near event horizon

We consider a dynamical black hole and investigate the late-time behavior of the near horizon geometry at late time. The black hole is defined as the region which is not contained in $J^-(\mathscr{I}^+)$ where $\mathscr{I}^+$ is the future null infinity. $J^-(S)$ is the chronological past connecting a set S by causal curves from S. The event horizon is the boundary of the black hole and it is a null hypersurface. See Ref. [2] for the precise definitions. Then, we introduce the Bondi-like (Gaussian null) coordinates $x^A = (u, r, x^a)$ near the event horizon as

$$ds^2 = g_{AB}dx^A dx^B = -Adu^2 + 2dudr + h_{ab}(dx^a + U^a du)(dx^b + U^b du), \tag{1}$$

where u is a time coordinate. In this coordinate, the horizon position is taken to be $r = 0$. We assume that a cross section of the event horizon is compact in the $u = \text{constant}$ hypersurface and its topology is S^2. x^a are coordinates on S^2. Since the event horizon is a null hypersurface, g_{uu} must vanish on the event horizon and $l = \partial/\partial u$ is the null geodesic generator on the event horizon. Furthermore we can choose the coordinate u so that $l_a = 0$ on the event horizon. Then, we have

$$A \,\hat{=}\, 0, \tag{2}$$

and

$$U^a \,\hat{=}\, 0, \tag{3}$$

where $\hat{=}$ means the evaluation on the event horizon $r = 0$.

To solve the vacuum Einstein equations, we formulate the initial value problem in the Bondi coordinates. For the convenience of our study in the following sections, we solve the Einstein equations as the evolution equations in the direction of r. Thus, the initial value of the metric should be set on the $r = \text{constant}$ surface. In this paper we take the event horizon $r = 0$ as the initial surface.

The evolution equations are given by

$$R_{rr} \;=\; -\frac{1}{2}(\log h)'' - \frac{1}{4}h^{ac}h^{bd}(h_{ab})'(h_{cd})' \;=\; 0, \tag{4}$$

$$R_{rB}h^{aB} \;=\; \frac{1}{2}U^{a''} + \frac{1}{2}h^{ac}h'_{bc}U^{b'} + \frac{1}{4}(\log h)'U^{a'} + \frac{1}{2}h^{ab}\bar{D}^c[h'_{bc} - h_{bc}(\log h)'] \;=\; 0 \tag{5}$$

and

$$\begin{aligned}
R_{AB}h^A_{\ a}h^B_{\ b} \;=\; & -\frac{1}{2}Ah''_{ab} - \frac{1}{2}A' h'_{ab} - (\dot{h}_{ab})' + {}^{(h)}R_{ab} + \frac{A}{2}h^{cd}h'_{ac}h'_{bd} + \frac{1}{2}h^{cd}(h'_{ac}\dot{h}_{bd} + h'_{bd}\dot{h}_{ac}) + \mathcal{L}_U h'_{ab} \\
& - \frac{1}{2}h'_{ac}(\bar{D}_bU^c + \bar{D}^cU_b) - \frac{1}{2}h'_{bc}(\bar{D}_aU^c + \bar{D}^cU_a) - \frac{1}{2}h_{ac}h_{bd}U^{c'}U^{d'} - \frac{1}{4}[(\dot{\log h}) - 2\bar{D}_aU^a]h'_{ab} \\
& - \frac{1}{4}(\log h)'(Ah'_{ab} + \dot{h}_{ab} - \bar{D}_aU_b - \bar{D}_bU_a) + \frac{1}{2}(h_{bc}\bar{D}_aU^{c'} + h_{ac}\bar{D}_bU^{c'}) \;=\; 0,
\end{aligned} \tag{6}$$

where the prime and dot denote the r and u derivative, respectively, $\bar{D}_a$ is a covariant derivative with h_{ab} and $h = \det h_{ab}$. Also, ${}^{(h)}R_{ab}$ is the Ricci tensor with respect to h_{ab}. The evolutions of the metric functions A, U^a and h_{ab} are determined by Eqs. (4), (5) and (6) completely. Note that the trace part of Eq. (6) gives us

$$\begin{aligned}
A'(\log h)' \;=\; & 2{}^{(h)}R - \frac{A}{2}[(\log h)']^2 - [(\dot{\log h}) - 2\bar{D}_aU^a](\log h)' - h_{ab}U^{a'}U^{b'} \\
& - A(\log h)'' - 2(\dot{\log h})' + 2(\bar{D}_aU^a)' + U^a\bar{D}_a(\log h)',
\end{aligned} \tag{7}$$

which is used as the evolution equation for A.

The other components of the Einstein equations, $R_{uu} = 0$, $R_{ur} = 0$ and $R_{ua} = 0$, are related to the above evolution equations by the Bianchi identities. Therefore, once they are satisfied at the initial surface $r = 0$, we do not need to solve them any more. In fact, if the evolution equations are satisfied, the Bianchi identities lead to

$$\begin{cases}
(\sqrt{h}R_{ua})' = \sqrt{h}R_{ur}, \\
(\sqrt{h}R^r_{\ u})' = -\sqrt{h}\bar{D}_a R^a_{\ u} - (\dot{\sqrt{h}})R_{ur}, \\
(\log h)'R_{ur} = 0,
\end{cases} \tag{8}$$

where $R^r{}_u = R_{uu} + AR_{ur} - U^a R_{ua}$ and $R^a{}_u = -U^a R_{ur} + h^{ab} R_{ub}$. Thus, the evolution equations guarantee that $R_{uu} = 0$, $R_{ur} = 0$ and $R_{ua} = 0$ will always be satisfied at $r \neq 0$ if $R_{uu} \hat{=} 0$, $R_{ur} \hat{=} 0$ and $R_{ua} \hat{=} 0$ at the initial surface $r = 0$. For convenience, at $r = 0$ we will use the following constraint equations:

$$R_{uu} \hat{=} -\frac{1}{2}(\ddot{\log h}) + \frac{A'}{4}(\dot{\log h}) - \frac{1}{4}h^{ac}h^{bd}\dot{h}_{ab}\dot{h}_{cd} = 0, \tag{9}$$

and

$$R^{ra} \hat{=} -\frac{1}{2}(\dot{U}^a)' - \frac{1}{4}(\dot{\log h})U^{a'} - \frac{1}{2}h^{ab}U^{c'}\dot{h}_{bc} - \frac{1}{2}\bar{D}^a A' + \frac{1}{2}h^{ab}\bar{D}^c \dot{h}_{bc} - \frac{1}{2}\bar{D}^a(\dot{\log h}) = 0 \tag{10}$$

[see Eqs. (A30) and (A31)]. Moreover, since A should vanish on the initial surface $r = 0$ [see Eq. (2)], the evolution equations Eq. (6) become singular on $r = 0$. Analyticity of h_{ab} on $r = 0$ gives us the regularity condition

$$-\frac{1}{2}A'h'_{ab} - \dot{h}'_{ab} + {}^{(h)}R_{ab} + \frac{1}{2}h^{cd}(h'_{ac}\dot{h}_{bd} + h'_{bd}\dot{h}_{ac})$$
$$-\frac{1}{2}h_{ac}h_{bd}U^{c'}U^{d'} - \frac{1}{4}(\dot{\log h})h'_{ab}$$
$$-\frac{1}{4}(\log h)'\dot{h}_{ab} + \frac{1}{2}(h_{bc}\bar{D}_a U^{c'} + h_{ac}\bar{D}_b U^{c'}) \hat{=} 0. \tag{11}$$

If we give $h_{ab}|_{r=0}$ on $r = 0$, we can determine $h'_{ab}|_{r=0}$, $U^{a'}|_{r=0}$, and $A'|_{r=0}$ by solving Eqs. (9), (10) and (11). As a result, we will solve the evolution equations (4), (5) and (6) using the above initial values on $r = 0$.

B. Some explicit solutions near event horizon

In this subsection, it is shown that we explicitly solve the constraint equations and evolution equations near the event horizon using power series expansion around $r = 0$. To do this, we expand the metric functions with r near the event horizon as

$$A = rA^{(1)} + \sum_{i \geq 2} r^i A^{(i)}, \tag{12}$$

$$U^a = rU^{(1)a} + \sum_{i \geq 2} r^i U^{(i)a}, \tag{13}$$

and

$$h_{ab} = h^{(0)}_{ab} + rh^{(1)}_{ab} + \sum_{i \geq 2} r^i h^{(i)}_{ab}, \tag{14}$$

where from the gauge conditions Eqs. (2) and (3), the expansions of A and U^a start from the first order of r. In the following, the tensor index of $h^{(i)}_{ab}$ and $U^{(i)a}$ is raised and lowered by $h^{(0)}_{ab}$. The trace and traceless parts are also taken by $h^{(0)}_{ab}$.

First let us solve the constraint equations and regularity conditions in order to determine initial values. On the initial surface $r = 0$, $h^{(0)}_{ab}$, $h^{(1)}_{ab}$, $A^{(1)}$ and $U^{(1)a}$ should be set on $r = 0$ as initial values. The constraint equations for these initial values are $R_{uu} \hat{=} 0$ and $R^{ra} \hat{=} 0$. Now $R_{uu} \hat{=} 0$ [Eq. (9)] is rewritten as

$$\ddot{h}^{(0)} - \frac{1}{2}A^{(1)}\dot{h}^{(0)} + \frac{1}{2}h^{(0)ac}h^{(0)bd}\dot{h}^{(0)}_{ab}\dot{h}^{(0)}_{cd} = 0, \tag{15}$$

where $h^{(0)} = \det h^{(0)}_{ab}$. Thus, $A^{(1)}$ should be given to satisfy this equation for given $h^{(0)}_{ab}$. Also, $R^{ra} \hat{=} 0$ [Eq. (10)] becomes

$$\dot{U}^{(1)a} = -D^a A^{(1)} + h^{(0)ac}D^b\dot{h}^{(0)}_{bc} - D^a(\log\dot{h}^{(0)}) - h^{(0)ac}U^{(1)b}\dot{h}^{(0)}_{bc} - \frac{1}{2}U^{(1)a}(\log\dot{h}^{(0)}), \tag{16}$$

where D_a is the covariant derivative with respect to $h_{ab}^{(0)}$. Then the initial value $U^{(1)a}$ is given to satisfy the above. The regularity condition Eq. (11) becomes

$$
-\dot{h}_{ab}^{(1)} - \frac{1}{2}A^{(1)}h_{ab}^{(1)} + {}^{(h^{(0)})}R_{ab} + \frac{1}{2}h^{(0)cd}(h_{ac}^{(1)}\dot{h}_{bd}^{(0)} + h_{bd}^{(1)}\dot{h}_{ac}^{(0)}) - \frac{1}{2}h_{ac}^{(0)}h_{bd}^{(0)}U^{(1)c}U^{(1)d}
$$
$$
- \frac{1}{4}(\log \dot{h}^{(0)})h_{ab}^{(1)} - \frac{1}{4}h^{(0)cd}h_{cd}^{(1)}\dot{h}_{ab}^{(0)} + \frac{1}{2}(D_aU_b^{(1)} + D_bU_a^{(1)}) = 0, \quad (17)
$$

where ${}^{(h^{(0)})}R_{ab}$ is the Ricci tensor of $h_{ab}^{(0)}$. We obtain $h_{ab}^{(1)}$ satisfying this equation. Hence, if we give $h_{ab}^{(0)}$ on the initial surface, the constraint equations and the regularity condition yield $h_{ab}^{(1)}$, $A^{(1)}$ and $U^{(1)a}$.

Next we solve the evolution equations. Equation (4) becomes near the event horizon as

$$
R_{rr} = -h^{(0)ab}h_{ab}^{(2)} + \frac{1}{4}(h_{ab}^{(1)})^2 + O(r) = 0. \tag{18}
$$

This equation gives us the trace part of $h_{ab}^{(2)}$ as

$$
h^{(0)ab}h_{ab}^{(2)} = \frac{1}{4}(h_{ab}^{(1)})^2. \tag{19}
$$

The evolution equation of U^a as $R_{rB}h^{aB} = 0$ [Eq. (5)] can be expanded as

$$
R_{rB}h^{aB} = U^{(2)a} + \frac{1}{2}h^{(0)ac}h_{bc}^{(1)}U^{(1)b} + \frac{1}{4}U^{(1)a}h^{(0)bc}h_{bc}^{(1)} + \frac{1}{2}h^{(1)ac}D^b\left(h_{bc}^{(1)} - h_{bc}^{(0)}h^{(0)de}h_{de}^{(1)}\right) + O(r). \tag{20}
$$

Then $U^{(2)a}$ is given by

$$
U^{(2)a} = -\frac{1}{2}h^{(0)ac}h_{bc}^{(1)}U^{(1)b} - \frac{1}{4}U^{(1)a}h^{(0)bc}h_{bc}^{(1)} - \frac{1}{2}h^{(1)ac}D^b\left(h_{bc}^{(1)} - h_{bc}^{(0)}h^{(0)de}h_{de}^{(1)}\right). \tag{21}
$$

In the same way, expanding the evolution equations Eq. (6) near the event horizon, we can obtain the traceless part $h_{\langle ab \rangle}^{(2)}$ and $A^{(2)}$ in terms of $h_{ab}^{(0)}, h_{ab}^{(1)}, U^{(1)a}$ and $A^{(1)}$. Note that $A^{(2)}$ is given by the trace part of Eq. (6), namely Eq. (7). However, we do not provide its explicit form because its form is very cumbersome.

Hence, we can determine $h_{ab}^{(2)}$, $U^{(2)a}$, and $A^{(2)}$ using the evolution equations. To determine the higher order quantities, $h_{ab}^{(i)}$, $U^{(i)a}$, and $A^{(i)}$ $(i > 2)$, we have to repeat the same procedure.

III. LATE-TIME SYMMETRY ON AND NEAR EVENT HORIZON

In this section, we show that there is late-time symmetry on the event horizon. Then we will extend it outside of black hole regions.

A. Late-time behaviors on event horizon

To investigate late-time behaviors of the event horizon, we introduce the expansion and shear of the null geodesic generator $l = \partial/\partial u$ of the event horizon. The expansion θ and shear σ_{ab} are defined as

$$
\sigma_{ab} + \frac{1}{2}\theta h_{ab}^{(0)} \; \hat{=} \; h_a{}^A h_b{}^B \nabla_A l_B
$$
$$
\hat{=} \; \frac{1}{2}\dot{h}_{ab}^{(0)}, \tag{22}
$$

where σ_{ab} is the traceless part of $\dot{h}_{ab}^{(0)}$ with respect to $h_{ab}^{(0)}$. Then we can rewrite Eq. (9), one of the constraint equations, using θ and σ_{ab} as

$$
\dot{\theta} - \frac{1}{2}A^{(1)}\theta = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab}. \tag{23}
$$

We can regard $A^{(1)}/2$ as a surface gravity of the black hole with respect to the time coordinate u because

$$l^A \nabla_A l^B \hat{=} \frac{A^{(1)}}{2} l^B \tag{24}$$

holds. Using the affine parameter w defined by

$$\frac{dw}{du} = \exp\left(\int^u \frac{A^{(1)}}{2} du' \right), \tag{25}$$

we can obtain the Raychaudhuri equation

$$\partial_w \theta_{(w)} = -\frac{1}{2}\theta^2_{(w)} - \sigma_{(w)ab}\sigma^{ab}_{(w)}, \tag{26}$$

where $\theta_{(w)}$ and $\sigma_{(w)ab}$ are expansion and shear with respect to w. We can see the relation between θ, σ_{ab} and $\theta_{(w)}$, $\sigma_{(w)ab}$ as

$$\theta = \theta_{(w)} \exp\left(\int^u \frac{A^{(1)}}{2} du' \right), \quad \sigma_{ab} = \sigma_{(w)ab} \exp\left(\int^u \frac{A^{(1)}}{2} du' \right). \tag{27}$$

Here we remember that the area law of the event horizon holds for spacetimes satisfying the null energy condition, that is, $\theta \geq 0$ and $\theta_{(w)} \geq 0$. Since $\sigma_{(w)ab}\sigma^{ab}_{(w)} \geq 0$, Eq. (26) implies the inequality

$$\partial_w \theta_{(w)} + \frac{1}{2}\theta^2_{(w)} \leq 0. \tag{28}$$

Then the integration over w gives us

$$\theta_{(w)} \leq \frac{1}{1/\theta_{(0)} + (w - w_0)/2} \to 0 \quad (\text{as } w \to \infty), \tag{29}$$

where we used the fact of $\theta_{(0)} = \theta_{(w)}(w = w_0) \geq 0$. In addition, Eq. (26) shows that the shear $\sigma_{(w)ab}$ should also vanish as $w \to \infty$. This is shown as a part of the proof of another theorem [10]. From now on, we assume that $w \to \infty$ corresponds to $u \to \infty$. Then, we see that θ and σ_{ab} should also vanish as $u \to \infty$. It is natural to assume that the cross section of the event horizon is compact. Then the vanishing of the expansion implies that the horizon area approaches a constant and finite value.

Altogether we see the behavior of the metric at late time as

$$\mathcal{L}_l g_{AB}|_{\text{horizon}} \to 0 \quad (u \to \infty). \tag{30}$$

Here we impose the following decaying condition on the event horizon for the metric:

$$\mathcal{L}_l g_{AB} \hat{=} O\left(\frac{1}{u^n} \right), \tag{31}$$

which explicitly means $\dot{h}^{(0)}_{ab} = O(u^{-n})$. This equation means that the null geodesic generator of the event horizon l should be an asymptotic Killing vector at late time ($u \to \infty$). Thus there is a late-time symmetry on the event horizon.

Since we consider the dynamics only near the event horizon, we cannot determine the decaying rate of the metric. However, the details of the decaying properties are not important for our argument, that is, our result does not depend on n. Note that it will be determined by the boundary conditions at a far region from the black holes as in Refs. [6–8].

It should be remembered that the horizon which satisfies this condition is called a slowly evolving horizon in Refs. [11, 12]. If the right-hand side of Eq. (31) vanishes, the event horizon will be identical to the isolated horizon [13].

B. Extension of late-time symmetry

The purpose of this subsection is to show that the decaying condition of Eq. (31) can be extended outside of the event horizon as

$$\mathcal{L}_l g_{AB} = O\left(\frac{1}{u^n} \right). \tag{32}$$

In the following we assume the analyticity of g_{AB}.

Under the presence of the analyticity of spacetimes, the above is equivalent with

$$(\mathcal{L}_n)^m \mathcal{L}_l g_{AB} \,\hat{=}\, O\left(\frac{1}{u^n}\right), \tag{33}$$

where $n = \partial/\partial r$ and $m = 0, 1, 2, \cdots$. Note that "$\hat{=}$" means the evaluation on the event horizon ($r = 0$) again.

First we will show the $m = 1$ case:

$$\mathcal{L}_n \mathcal{L}_l g_{AB} \,\hat{=}\, O\left(\frac{1}{u^n}\right). \tag{34}$$

Substituting the explicit form of g_{AB} to the above, we rewrite Eq. (34) as

$$-\mathcal{L}_n \mathcal{L}_l (A - U_a U^a)(du)_A (du)_B + 2\mathcal{L}_n \mathcal{L}_l U_a (du)_{(A}(dx^a)_{B)} + \mathcal{L}_n \mathcal{L}_l h_{ab}(dx^a)_A (dx^b)_B \,\hat{=}\, O\left(\frac{1}{u^n}\right). \tag{35}$$

Using Eqs. (12), (13) and (14), the above will be equivalent with

$$\mathcal{L}_l A^{(1)} = O\left(\frac{1}{u^n}\right), \tag{36}$$

$$\mathcal{L}_l U^{(1)a} = O\left(\frac{1}{u^n}\right), \tag{37}$$

and

$$\mathcal{L}_l h_{ab}^{(1)} = O\left(\frac{1}{u^n}\right). \tag{38}$$

Let us examine these conditions. First, we focus on Eq. (36). As a result, using the gauge freedom, we can show that the slightly strong condition, $\mathcal{L}_l A^{(1)} = O\left(1/u^{n+1}\right)$, holds. To see this, we decompose $A^{(1)}$ into the u-independent term and others as

$$A^{(1)} = A_0^{(1)}(x^a) + \tilde{A}^{(1)}(u, x^a). \tag{39}$$

As shown in Ref. [14], we can choose the gauge so that $A_0^{(1)}$ is a constant. Furthermore, using the residual gauge, we can take $\tilde{A}^{(1)} = O(1/u^n)$. Then, using the gauge freedom in our coordinate, we can make $A^{(1)}$ satisfy stronger decaying property as

$$\mathcal{L}_l A^{(1)} = O\left(\frac{1}{u^{n+1}}\right). \tag{40}$$

For the details, see Appendix C. Of course, Eq. (36) is satisfied.

Now $U^{(1)a}$ should satisfy the constraint equation of Eq. (16) as

$$\mathcal{L}_l U^{(1)a} = -D^a \tilde{A}^{(1)} + h^{(0)ac} D^b \dot{h}_{bc}^{(0)} - D^a(\log \dot{h}^{(0)}) - h^{(0)ac} U^{(1)b} \dot{h}_{bc}^{(0)} - \frac{1}{2} U^{(1)a}(\log \dot{h}^{(0)}). \tag{41}$$

Together with Eqs. (31) and (40), we can see that Eq. (37) holds from the above.

Furthermore, $h_{ab}^{(1)}$ satisfy the following equation [see Eq. (17)] as a regularity condition

$$\dot{h}_{ab}^{(1)} + \frac{1}{2} A^{(1)} h_{ab}^{(1)} = {}^{(h^{(0)})} R_{ab} + \frac{1}{2} h^{(0)cd}(h_{ac}^{(1)} \dot{h}_{bd}^{(0)} + h_{bd}^{(1)} \dot{h}_{ac}^{(0)}) - \frac{1}{2} h_{ac}^{(0)} h_{bd}^{(0)} U^{(1)c} U^{(1)d}$$
$$- \frac{1}{4}(\log \dot{h}^{(0)}) h_{ab}^{(1)} - \frac{1}{4} h^{(0)cd} h_{cd}^{(1)} \dot{h}_{ab}^{(0)} + \frac{1}{2}(D_a U_b^{(1)} + D_b U_a^{(1)}), \tag{42}$$

Then multiplying $\mathcal{L}_l$ to the above and using Eq. (16), we can see

$$\frac{1}{2} A^{(1)} \mathcal{L}_l h_{ab}^{(1)} = \mathcal{L}_l {}^{(h^{(0)})} R_{ab} + O(u^{-n}) \tag{43}$$

holds. In the above, the higher-order terms like $\mathcal{L}_l^2 h_{ab}^{(1)}, (\mathcal{L}_l h_{ab}^{(0)})^2, (\mathcal{L}_l h_{ab}^{(1)})^2, \mathcal{L}_l h_{ab}^{(0)} \mathcal{L}_l h_{ab}^{(1)}$ and so on are contained in $O(u^{-n})$. Thus Eq. (31) tells us that $\mathcal{L}_l h_{ab}^{(1)} = O(u^{-n})$ holds. As a consequence, we can show the $m = 1$ case of Eq. (33).

For the $m > 1$ cases of Eq. (33), we perform the same procedure inductively. Let $m > 1$ be an integer and assume that the metric satisfies

$$(\mathcal{L}_n)^k \mathcal{L}_l g_{AB} \hat{=} O\left(\frac{1}{u^n}\right) \tag{44}$$

for all $k(< m)$. Then $(\mathcal{L}_n)^m \mathcal{L}_l g_{AB} \hat{=} O(u^{-n})$ is equivalent with

$$\mathcal{L}_l A^{(m)} = O\left(\frac{1}{u^n}\right), \tag{45}$$

$$\mathcal{L}_l U^{(m)a} = O\left(\frac{1}{u^n}\right), \tag{46}$$

and

$$\mathcal{L}_l h_{ab}^{(m)} = O\left(\frac{1}{u^n}\right). \tag{47}$$

Since $U^{(m)a}$ is written by $U^{(1)a}, A^{(1)}, h_{ab}^{(0)}, \cdots, h_{ab}^{(m-1)}$ from Eq. (5) like Eq. (21), the induction assumption

$$\mathcal{L}_l h_{ab}^{(i)} = O\left(\frac{1}{u^n}\right) \quad (i \leq m - 1) \tag{48}$$

immediately shows us that Eq. (46) holds.

From Eq. (7), $A^{(m)}$ is written by $U^{(1)}, A^{(1)}, h_{ab}^{(i)}(i < m)$ and the trace part of $h_{ab}^{(m)}$. Since the trace part of $h_{ab}^{(m)}$ is written by $U^{(1)}, A^{(1)}$ and $h_{ab}^{(i)}$ $(i < m)$ through Eq. (4), $A^{(m)}$ is written only by $U^{(1)}, A^{(1)}, h_{ab}^{(i)}$ $(i < m)$ in the end. Then, by the assumption of the induction, Eq. (45) holds. Next, the evolution equation for $h_{ab}^{(m)}$ is described by Eq. (6). Expanding Eq. (6) near the event horizon and multiplying, $(\mathcal{L}_n)^{m-1}$, Eq. (6) becomes the following form on the event horizon

$$\dot{h}_{ab}^{(m)} + \frac{1}{2} A^{(1)} h_{ab}^{(m)} = F_{ab}, \tag{49}$$

where F_{ab} is a function of $h_{ab}^{(i)}$ $(i \leq m)$, $\dot{h}_{ab}^{(i)}$ $(i < m)$ and so on. Acting $\mathcal{L}_l$ to Eq. (49), then, we can see that $(1/2) A^{(1)} \dot{h}_{ab}^{(m)} = \dot{F}_{ab} \sim \dot{h}_{ab}^{(j)} = O(1/u^n)$ for $j < m$. Thus Eq. (47) holds.

Now we can confirm that the induction loop is closed. Then we can show that Eq. (33), equivalently Eq. (32) holds if the spacetime is real analytic. Therefore we can show that the null geodesic generator l of the event horizon is the asymptotic Killing vector at late time in the sense of Eq. (32).

IV. SUMMARY AND DISCUSSION

We have confirmed that the expansion and shear of the event horizon should decay at late time in the vacuum spacetimes. Then, assuming the compactness of the cross sections of the event horizon, the null geodesic generators on the horizon give us an asymptotic Killing vector l at late time. This means that the horizon has late-time symmetry. By solving the Einstein equations, then, we have found that this late-time symmetry can be extended outside of the black holes. Therefore, at late time, there is the asymptotic symmetry outside of black holes.

If the black hole rotates and there is another asymptotic Killing vector at late time, k, which will be a stationary Killing vector at a far distance and spacelike near the horizon, $k - l$ is also an asymptotic Killing vector expected to correspond to axisymmetry. In this sense, we would expect that the rigidity holds in gravitational collapse at late time. In these discussions, we assume the compactness of the event horizon. Thus this result cannot be applied to other null hypersurfaces which do not have a compact cross section.

There is a remaining issue. In the proof of the rigidity theorem, the exact stationarity does show that the null geodesic generators of the horizon are Killing orbits. On the other hand, our argument could show us that the null geodesic generators of the horizon is a Killing orbit without assuming the presence of asymptotically stationary Killing vectors. It is likely that this difference suggests the existence of important and new points in black hole physics.

Appendix A: Einstein equations near event horizon

In Appendix A, using the suitable variables, we will show the derivations of the Einstein equations near the event horizon.

1. $(n+1)$ decomposition

First we describe the formula of the $(n+1)$ decomposition. The metric can be written as

$$g_{AB} = \epsilon n_A n_B + \gamma_{AB}, \tag{A1}$$

where γ_{AB} is an n-dimensional induced metric. n_A is the unit normal vector with $n_A n^A = \epsilon = +1$ (n^A: spacelike) or -1 (n^A: timelike).

We define the extrinsic curvature as

$$K_{AB} = \frac{1}{2}\mathcal{L}_n \gamma_{AB}. \tag{A2}$$

Now n_A can be written as $n_A = \epsilon N(d\Omega)_A$, where Ω is a function which describes the hypersurface as $\Omega = $ const. and N is the "lapse" function. Then the Riemann tensor is decomposed into

$$R_{EFGH}\gamma_A{}^E\gamma_B{}^F\gamma_C{}^G\gamma_D{}^H = {}^{(\gamma)}R_{ABCD} - \epsilon K_{AC}K_{BD} + \epsilon K_{AD}K_{BC}, \tag{A3}$$

$$R_{EFGD}\gamma_A{}^E\gamma_B{}^F\gamma_C{}^G n^D = \nabla_A K_{BC} - \nabla_B K_{AC}, \tag{A4}$$

$$R_{ACBD}n^C n^D = -\mathcal{L}_n K_{AB} + K_{AC}K_B{}^C - \epsilon\frac{1}{N}\nabla_A\nabla_B N, \tag{A5}$$

where ∇_A denotes the covariant derivative with respect to γ_{AB}.

The Ricci tensor is decomposed into

$$R_{AB}n^A n^B = -\mathcal{L}_n K - K_{AB}K^{AB} - \epsilon\frac{1}{N}\nabla^2 N, \tag{A6}$$

$$R_{AC}n^A\gamma_B{}^C = \nabla^A K_{AB} - \nabla_B K, \tag{A7}$$

$$R_{CD}\gamma_A{}^C\gamma_B{}^D = {}^{(\gamma)}R_{AB} - \epsilon\mathcal{L}_n K_{AB} - \epsilon K K_{AB} + 2\epsilon K_{AC}K_B{}^C - \frac{1}{N}\nabla_A\nabla_B N. \tag{A8}$$

The Ricci scalar is written as

$$\begin{aligned}R =& {}^{(\gamma)}R - 2\epsilon\mathcal{L}_n K - \epsilon K^2 - \epsilon K_{AB}K^{AB} - \frac{2}{N}\nabla^2 N \\ =& {}^{(\gamma)}R + \epsilon K^2 - \epsilon K_{AB}K^{AB} - \frac{2}{N}\nabla^2 N - 2\epsilon\nabla_A(Kn^A).\end{aligned} \tag{A9}$$

The components of the Einstein tensor are

$$G_{AB}n^A n^B = \frac{1}{2}(-\epsilon{}^{(\gamma)}R + K^2 - K_{AB}K^{AB}), \tag{A10}$$

$$G_{AC}n^A\gamma_B{}^C = \nabla^A K_{AB} - \nabla_B K, \tag{A11}$$

$$G_{CD}\gamma_A{}^C\gamma_B{}^D =^{(\gamma)}G_{AB} - \epsilon K K_{AB} + 2\epsilon K_{AC}K_B{}^C + \frac{\epsilon}{2}\gamma_{AB}(K_{CD}K^{CD} + K^2)$$
$$- \epsilon\mathcal{L}_n K_{AB} + \epsilon\gamma_{AB}\mathcal{L}_n K - \frac{1}{N}\nabla_A\nabla_B N + \frac{1}{N}\gamma_{AB}\nabla^2 N. \tag{A12}$$

2. Einstein equations

We apply the $(n+1)$ decomposition presented in the previous section to the r-constant surface in our current four dimensional metric form:

$$ds^2 = -Adu^2 + 2dudr + h_{ab}(dx^a + U^a du)(dx^b + U^b du). \tag{A13}$$

We express the above in the following form

$$ds^2 = N^2 dr^2 + q_{\mu\nu}(dx^\mu + N^\mu dr)(dx^\nu + N^\nu dr), \tag{A14}$$

where

$$N^2 = \frac{1}{A}, \tag{A15}$$

$$N^u = -\frac{1}{A}, \tag{A16}$$

$$N^a = \frac{1}{A}U^a. \tag{A17}$$

$q_{\mu\nu}$ is the induced metric on the r-const. hypersurface as

$$q_{\mu\nu} = \begin{pmatrix} -A + U^a U_a & U_b \\ U_a & h_{ab} \end{pmatrix}. \tag{A18}$$

Note that the Latin indices $a, b, ...$ and the Greek indices $\mu, \nu, ..$ are raised and lowered by h_{ab} and $q_{\mu\nu}$ respectively. The unit normal vector to the r-const. hypersurface is given by $m_A = N(dr)_A$ and $m^A = N^{-1}(\partial_r - N^\mu\partial_\mu)^A$.

The extrinsic curvature on the r-const. hypersurface is defined as

$$K_{\mu\nu} = \frac{1}{2}\mathcal{L}_m q_{\mu\nu} = \frac{1}{2N}(\partial_r q_{\mu\nu} - \mathcal{D}_\mu N_\nu - \mathcal{D}_\nu N_\mu), \tag{A19}$$

where $\mathcal{D}_\mu$ is the covariant derivative with respect to $q_{\mu\nu}$.

We rewrite the induced metric as

$$q_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 du^2 + h_{ab}(dx^a + \beta^a du)(dx^b + \beta^b du), \tag{A20}$$

where

$$\alpha^2 = A, \ \beta^a = U^a. \tag{A21}$$

The timelike unit vector to the u-const. surface is given by $u = -\alpha du$ and $u^\mu = \alpha^{-1}(\partial_u - \beta^a\partial_a)^\mu$.

The extrinsic curvature on the u-const. surface is given by

$$k_{ab} = \frac{1}{2}\mathcal{L}_u h_{ab} = \frac{1}{2\alpha}(\partial_u h_{ab} - \bar{D}_a\beta_b - \bar{D}_b\beta_a), \tag{A22}$$

where $\bar{D}_a$ is the covariant derivative with respect to h_{ab}.

We introduce the following quantities for later convenience

$$\Theta \equiv K_{\mu\nu}u^\mu u^\nu = -\frac{1}{N}\partial_r(\log\alpha) - \mathcal{L}_u\log N, \tag{A23}$$

$$\rho^a \equiv K^a_\mu u^\mu = \frac{1}{2}\partial_r\beta^a + \bar{D}^a \log\alpha, \tag{A24}$$

$$\kappa_{ab} \equiv K_{cd}h_a{}^c h_b{}^d = \frac{\alpha}{2}\partial_r h_{ab} + k_{ab}, \tag{A25}$$

and

$$\kappa = \kappa_{ab}h^{ab} = \frac{\alpha}{2}(\log h)^{'} + k, \tag{A26}$$

where $\rho_\mu u^\mu = 0 = \kappa_{\mu\nu}u^\mu$, $h = \det h_{ab}$ and $k = k_{ab}h^{ab}$. The prime denotes the r differentiation. Then $K_{\mu\nu}$ can be written as

$$K_{\mu\nu} = \Theta u_\mu u_\nu - 2\rho_{(\mu}u_{\nu)} + \kappa_{\mu\nu}. \tag{A27}$$

Using these quantities, we can decompose the four dimensional Ricci tensor R_{AB} into the quantities on two dimensional space:

$$\begin{aligned} R_{AB}m^A m^B = {} & \frac{1}{N}(\Theta - \kappa)^{'} + \mathcal{L}_u(\Theta - \kappa) - \Theta^2 + 2\rho_a\rho^a - \kappa_{ab}\kappa^{ab} \\ & + \frac{1}{N}(\mathcal{L}_u\mathcal{L}_u N + k\mathcal{L}_u N - \bar{D}^2 N - \bar{D}^a N \bar{D}_a \log\alpha), \end{aligned} \tag{A28}$$

$$R_{AB}m^A q^B{}_C u^C = -\mathcal{L}_u\kappa + \bar{D}^a\rho_a - k_{ab}\kappa^{ab} + 2\rho^a\bar{D}_a\log\alpha - \Theta k, \tag{A29}$$

$$\begin{aligned} R_{AB}q^A{}_C q^B{}_D u^C u^D = {} & -\frac{1}{N}\Theta^{'} - \mathcal{L}_u\Theta + \Theta^2 - \Theta\kappa - 2\rho^a\rho_a - 2\rho^a\bar{D}_a\log\frac{N}{\alpha} \\ & + \bar{D}^a\log\alpha\bar{D}_a\log N + \frac{1}{\alpha}\bar{D}^2\alpha - \mathcal{L}_u k - k_{ab}k^{ab} - \frac{1}{N}\mathcal{L}_u\mathcal{L}_u N, \end{aligned} \tag{A30}$$

$$R_{AB}m^A h^{Ba} = \Theta\bar{D}^a\log\alpha - 2\rho_b k^{ab} - k\rho^a + \bar{D}_b\kappa^{ab} - \bar{D}^a\kappa + \bar{D}^a\Theta + \kappa^{ab}\bar{D}_b\log\alpha - \frac{1}{\alpha}(\partial_u\rho^a - \mathcal{L}_\beta\rho^a), \tag{A31}$$

$$\begin{aligned} R_{AB}q^A{}_C u^C h^{Bb} = {} & \bar{D}_a k^{ab} - \bar{D}^b k - \rho^b\kappa - 2\rho_a\kappa^{ab} - \frac{1}{N}\bar{D}^b\mathcal{L}_u N + k^{ab}\bar{D}_a\log N - \frac{1}{N}(\rho^b)^{'} \\ & - \Theta\bar{D}^b\log\frac{N}{\alpha} - \kappa^{ab}\bar{D}_a\log\frac{N}{\alpha} - \frac{1}{\alpha}(\partial_u\rho^b - \mathcal{L}_\beta\rho^b), \end{aligned} \tag{A32}$$

and

$$\begin{aligned} R_{AB}h^A{}_a h^B{}_b = {} & {}^{(h)}R_{ab} + \mathcal{L}_u k_{ab} + k k_{ab} - 2k_{ac}k_b{}^c - \frac{1}{\alpha}\bar{D}_a\bar{D}_b\alpha - \frac{1}{N}\kappa^{'}_{ab} - \frac{1}{\alpha}\dot{\kappa}_{ab} + \frac{1}{\alpha}\mathcal{L}_\beta\kappa_{ab} - 2\rho_{(a}\bar{D}_{b)}\log\frac{N}{\alpha} \\ & + (\Theta - \kappa)\kappa_{ab} - 2\rho_a\rho_b + 2\kappa_{ac}\kappa_b{}^c - \frac{1}{N}\bar{D}_a\bar{D}_b N + k_{ab}\mathcal{L}_u\log N, \end{aligned} \tag{A33}$$

where the dot denotes ∂_u. The vacuum Einstein equation is given by $R_{AB} = 0$.

Appendix B: Explicit form of Einstein equations

In Appendix B, we describe the components of the Einstein equations in terms of the metric functions explicitly. Using

$$u = \alpha^{-1}(l - U^a\partial_a) = \alpha^{-1}(\partial_u - U^a\partial_a), \tag{B1}$$

$$\Theta = -(A^{1/2})^{'} + \frac{1}{2}\mathcal{L}_u\log A, \tag{B2}$$

$$\rho^a = \frac{1}{2}U^{a'} + \frac{1}{2}\bar{D}^a \log A, \tag{B3}$$

and

$$\kappa_{ab} = \frac{A^{1/2}}{2}h'_{ab} + \frac{1}{2A^{1/2}}(\dot{h}_{ab} - \bar{D}_a U_b - \bar{D}_b U_a), \tag{B4}$$

we can rewrite the Einstein equation $R_{AB} = 0$ in terms of our metric form. We will not provide all components of the Einstein equation explicitly.

For example, $R_{AB}h^A{}_a h^B{}_b = 0$ becomes

$$
\begin{aligned}
\mathcal{L}_l h'_{ab} + \frac{1}{2}A'h'_{ab} = {}^{(h)}R_{ab} &+ \frac{A}{2}h^{cd}h'_{ac}h'_{bd} + \frac{1}{2}h^{cd}(h'_{ac}\dot{h}_{bd} + h'_{bd}\dot{h}_{ac}) - \frac{1}{2}h'_{ac}(\bar{D}_b U^c + \bar{D}^c U_b) + \mathcal{L}_U h'_{ab} \\
&- \frac{1}{2}h'_{bc}(\bar{D}_a U^c + \bar{D}^c U_a) - \frac{1}{2}h_{ac}h_{bd}U^{c'}U^{d'} - \frac{1}{4}[(\log h)^{\cdot} - 2\bar{D}_a U^a]h'_{ab} \\
&- \frac{1}{4}(\log h)'(Ah'_{ab} + \dot{h}_{ab} - \bar{D}_a U_b - \bar{D}_b U_a) - \frac{1}{2}Ah''_{ab} + \frac{1}{2}(h_{bc}\bar{D}_a U^{c'} + h_{ac}\bar{D}_b U^{c'}). \tag{B5}
\end{aligned}
$$

This determines the evolutions of h_{ab}. For $R_{AB}m^A h^{Ba} = 0$, we have

$$
\begin{aligned}
\mathcal{L}_l U^{a'} = {}&-\bar{D}^a A' - h^{ac}U^{b'}(\dot{h}_{bc} - \bar{D}_a U_b - \bar{D}_b U_a) + h^{ab}\bar{D}^c(Ah'_{bc} + \dot{h}_{bc} - \bar{D}_b U_c - \bar{D}_c U_b) - A\bar{D}^a(\log h)' \\
&- \frac{1}{2}U^{a'}[(\log h)^{\cdot} - 2\bar{D}_a U^a] - \frac{1}{2}(\log h)'\bar{D}^a A - \bar{D}^a[(\log h)^{\cdot} - 2\bar{D}_b U^b] + \mathcal{L}_U U^{a'}. \tag{B6}
\end{aligned}
$$

Appendix C: The gauge issue for $A^{(1)}$ [Eq. (40)]

In Appendix C we will show the presence of the gauge where Eq. (40) is satisfied. In our coordinate $x^A = (u, r, x^a)$, the metric can be written as

$$ds^2 = -Adu^2 + 2dudr + h_{ab}(dx^a + U^a du)(dx^b + U^b du). \tag{C1}$$

The components of the metric are expanded near the event horizon $(r = 0)$ as

$$A = rA^{(1)} + O(r^2), \tag{C2}$$

$$U^a = rU^{(1)a} + O(r^2) \tag{C3}$$

and

$$h_{ab} = h_{ab}^{(0)} + O(r). \tag{C4}$$

Here $A^{(1)}$ can be decomposed as

$$A^{(1)} = A_0^{(1)} + \tilde{A}^{(1)}(u, x^a), \tag{C5}$$

where $A_0^{(1)}$ is set to be a constant as shown in Ref. [14].

When we consider the gauge transformation $x^A \to x^A + \xi^A$, the metric is transformed as

$$g_{AB} \to g_{AB} + \mathcal{L}_\xi g_{AB} \equiv g_{AB} + \delta g_{AB}. \tag{C6}$$

To keep our gauge, the following conditions will be imposed:

$$
\begin{aligned}
\delta g_{ur} = 0\,, \ \delta g_{rr} = 0\,, \ \delta g_{ra} = 0\,, \\
\delta g_{uu} = O(r)\,, \ \delta g_{ua} = O(r)\,, \ \delta g_{ab} = O(1). \tag{C7}
\end{aligned}
$$

From $\delta g_{rr} = 2\partial_r \xi^u = 0$, we have $\xi^u = f(u, x^a)$. Since δg_{ra} and δg_{ur} are given by

$$\delta g_{ra} = \partial_a \xi^u + U_a \partial_r \xi^u + h_{ab}\partial_r \xi^b, \tag{C8}$$

$$\delta g_{ur} = (-A + U^a U_a)\partial_r \xi^u + \partial_r \xi^r + \partial_u \xi^u + U_a \partial_r \xi^a, \tag{C9}$$

$\delta g_{ra} = 0$ and $\delta g_{ur} = 0$ lead to

$$\xi^r = -r\partial_u f + \partial_a f \int^r U^a dr', \quad \xi^a = -\partial_b f \int^r h^{ab} dr'. \tag{C10}$$

Then δg_{uu} becomes

$$\delta g_{uu} = r[-\partial_u(fA^{(1)}) - \partial_u^2 f] + O(r^2), \tag{C11}$$

where we used Eqs. (C2) and (C3). This means that $A^{(1)}$ is transformed under the gauge transformation as

$$A^{(1)} \to A^{(1)} + \partial_u(fA^{(1)}) + \partial_u^2 f. \tag{C12}$$

Thus if we choose f as

$$f = -\frac{1}{A_0^{(1)}} \int^u du' \tilde{A}^{(1)}, \tag{C13}$$

$A^{(1)}$ is transformed as

$$A^{(1)} \to \bar{A}^{(1)} = A_0^{(1)} - \frac{1}{A_0^{(1)}} [\partial_u \tilde{A}^{(1)} + \partial_u(f\tilde{A}^{(1)})]. \tag{C14}$$

Let assume that $\tilde{A}^{(1)}$ decays as $u \to \infty$. For the moment, we write $\tilde{A}^{(1)} = O(1/u^m)$, where m is an integer. If $m \geq n$, Eq. (40) is already satisfied. Therefore, we suppose that m is smaller than n.

In the current gauge transformation, the transformed $\bar{A}^{(1)}$ satisfies

$$\bar{A}^{(1)} = A_0^{(1)} + O(1/u^{m+1}). \tag{C15}$$

Repeating this procedure, we can always choose the gauge satisfying

$$\mathcal{L}_l A^{(1)} = O\left(\frac{1}{u^{n+1}}\right). \tag{C16}$$

If one wishes, one can continue the same procedure and then achieve an arbitrary faster decaying rate. But, the above is enough for our current purpose.

[1] W. Israel, Phys. Rev. **164**, 1776 (1967); B. Carter, Phys. Rev. Lett. **26**, 331 (1971); S. W. Hawking, Commun. Math. Phys. **25**, 152 (1972); D. C. Robinson, Phys. Rev. Lett. **34**, 905 (1975); P. O. Mazur, J. Phys. **A15**, 3173 (1982); For a review, see M. Heusler, *Black Hole Uniqueness Theorems*, (Cambridge University Press, London, 1996); P. O. Mazur, hep-th/0101012; G. L. Bunting, PhD thesis, Univ. of New England, Armidale (1983).
[2] S. W. Hawking and G. F. R. Ellis, *The Large scale structure of space-time*, (Cambridge Univ. Press, Cambridge, 1973).
[3] S. W. Hawking, Commun. Math. Phys. **25**, 152-166 (1972).
[4] H. Friedrich, I. Racz, R. M. Wald, Commun. Math. Phys. **204**, 691-707 (1999).
[5] I. Racz, Class. Quant. Grav. **17**, 153-178 (2000).
[6] C. Gundlach, R. H. Price, J. Pullin, Phys. Rev. **D49**, 883-889 (1994).
[7] L. Barack, A. Ori, Phys. Rev. Lett. **82**, 4388 (1999).
[8] L. Barack, A. Ori, Phys. Rev. **D60**, 124005 (1999).
[9] K. Tanabe, S. Kinoshita, T. Shiromizu, Phys. Rev. **D84**, 044055 (2011).
[10] S. A. Hayward, T. Shiromizu, K. -i. Nakao, Phys. Rev. **D49**, 5080-5085 (1994).
[11] I. Booth, S. Fairhurst, Phys. Rev. Lett. **92**, 011102 (2004).
[12] I. Booth, S. Fairhurst, Phys. Rev. **D75**, 084019 (2007).
[13] A. Ashtekar, B. Krishnan, Living Rev. Rel. **7**, 10 (2004).
[14] S. Hollands, A. Ishibashi, R. M. Wald, Commun. Math. Phys. **271**, 699-722 (2007).